# DrawLine

We will loop over one axis and draw the corresponding points on the other axis. We loop over the longer axis.

Formally: define . Loop over if otherwise loop over

We will always assume that if we loop over then and if we loop over then . This simplifies the equations and can be forced simply by swapping the two points whenever the condition is violated.

## Line equation:

## Slope equation:

## DrawLine v1:

From Line equation:

|  |  |
| --- | --- |
| Loop over | Loop over |
|  |  |

## DrawLine v2:

Given:

|  |  |
| --- | --- |
|  | Using #1 |
|  | By Cancelation |
|  | Using #2 |
|  | By Cancelation |
|  | Reordering |

Using the similar reasoning for , we arrive at

|  |  |
| --- | --- |
| Loop over | Loop over |
|  |  |

## DrawLine v3:

Notice that during this analysis we assume we are looping over , the proof for looping over is very similar but not written extensively here, however the results are written in the tables.

Subproblem: you have point and you need to deduce whether the point is above/below the line and whether it is to the right/left of the line

|  |  |
| --- | --- |
|  | Line equation |
|  | Expansion |
|  | Substituting using slope equation |
|  | Multiplying by |
|  | Reordering |

This is now in the form , where and .

So, if we set and plug in a value we can deduce the relation between the point and the line as follows:

and are the same sign iff is above the line, otherwise it is below the line.

Similarly, and are the same sign iff is to the right of the line, otherwise it is to the left of the line.

If and are the same sign, it means that increasing increases , so the point becomes farther from the line as it moves upwards, therefore it is above the line in the first place.

If and are of a different sign, it means that decreasing increases , so the point becomes farther from the line as it moves downwards, therefore it is below the line in the first place.

Similar reasoning can be deduced on and for the rightwards and leftwards relation between the point and the line.

Let’s assume we loop over and

If we drew a point the next point we draw should either be or . We will draw the point that is closer to the line. To know which point is closer, we will check the midpoint between them, if the midpoint is above the line, the first is closer, if the midpoint is below the line, the second is closer. The midpoint is , we will use as we demonstrated to check the relation between the point and the line.

Since we only care about the sign of , we can multiply it by so we don’t have to use a double to store the midpoint.

Set

So, set

To know if and are the same sign we check whether is or not. so we check

We will simply set:

To remove the constraint . We assign:

the next point we draw should either be or . To know which point is closer, we will check the midpoint between them, midpoint being .

This changes the equation of to:

However, which point is closer now also depends on . If diff is positive, it is as stated before, if it is negative, then the opposite is true: the second point is closer if the midpoint is above the line. In another word, the equation we set for need to be reversed if diff is negative. To do that, we simply multiply the condition by diff so the sign is flipped if it needs to be flipped:

The exact same procedure will be done for looping over y:

Notice that the condition doesn’t have a negative sign because we compare instead of , and is just not .

|  |  |
| --- | --- |
| Loop over | Loop over |
|  |  |

Our final optimization is as follows: instead of calculating and each step, we will calculate it from the previous step as follows:

Recall that and is the difference between the current step and the previous step. In our analysis, we loop over , so while is either or .

At the start, we need to find in the initial step:

All the above reasoning can be done for looping over too, doing both will result in the following formulas:

|  |  |  |  |
| --- | --- | --- | --- |
| Loop over | | Loop over | |
| Using  We get: | | **Using**  **We get:** | |
| If diff is added | **If diff is not added** | **If diff is added** | **If diff is not added** |
|  |  |  |  |